

Contributions of the Landscapes of Investigation to critical learning in the context of Field Education

Abstract: This paper is an excerpt from a master's research project based on Critical Mathematics Education, Field Education and Financial Education. One of the aims of this research is to understand how projects based on landscapes of investigation can contribute to critical learning in field education. As a methodology, we carried out participant research, developing a landscape of investigation in a municipal school in the interior of Bahia. Based on the participant research, we understand, in the light of Paulo Freire, that, in field education, landscapes of investigation in mathematics class provide the exercise of citizenship by democratizing the possibility of the naturalness of mathematics as a condition of being in the world.

Keywords: Critical Mathematics Education. Field Education. Financial Education. Landscapes of Investigation. Critical Learning.

Aportes de los Escenarios de Investigación al aprendizaje crítico en el contexto de la Educación Rural

Resumen: Este artículo es un extracto de una investigación de maestría que tiene como base Educación Matemática Crítica, Educación Rural y Educación Financiera. Uno de los objetivos de esta investigación es comprender cómo proyectos basados en escenarios de investigación pueden contribuir al aprendizaje crítico en Educación Rural. Como metodología se realizó una investigación participante con el desarrollo de un escenario de investigación en una escuela municipal del interior de Bahia. Al realizar la investigación participativa, entendemos, a luz de Paulo Freire, que, en la Educación Rural, los escenarios de investigación en las clases de matemáticas propician el ejercicio de la ciudadanía al democratizar la posibilidad de la naturalidad de las matemáticas como condición de estar en el mundo.

Palabras clave: Educación Matemática Crítica. Educación Rural. Educación Financiera. Escenarios de Investigación. Aprendizaje Crítico.


Contribuições dos Cenários para Investigação para uma aprendizagem crítica no contexto da Educação do Campo

Resumo: Este artigo é um recorte de uma pesquisa de mestrado que se fundamenta na Educação Matemática Crítica, na Educação do Campo e na Educação Financeira. Um dos objetivos da referida pesquisa é compreender como projetos calcados nos cenários para investigação podem contribuir para uma aprendizagem crítica em uma Educação do Campo. Como metodologia, realizamos uma pesquisa participante, desenvolvendo um cenário para investigação em uma escola municipal do interior baiano. A partir da pesquisa participante, compreendemos, à luz de Paulo Freire, que, na Educação do Campo, os cenários para investigação na aula de Matemática proporcionam o exercício da cidadania ao democratizar a possibilidade da naturalidade da Matemática como uma condição de estar no mundo.

Palavras-chave: Educação Matemática Crítica. Educação do Campo. Educação Financeira. Cenários para Investigação. Aprendizagem Crítica.

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Received • 29/03/2024

Accepted • 15/05/2024

Published • 20/08/2024

Article

1 Introduction

This paper¹ aims to present how Critical Mathematics Education (CME) can contribute to critical learning in the context of field education. To this end, a landscape of investigation was developed with reference to one of the peasant activities carried out by the students' families for subsistence: the marketing of products from their agricultural production and also the resale of products at the city's open market.

From this perspective, this landscape of investigation involved working with Financial Education with students from a 3rd grade class in the Early Years of Primary School, in a school located in the rural area of the municipality of Utinga, which is part of the Chapada Diamantina region in the state of Bahia.

The analyses and results presented in this paper help to validate the hypothesis of the aforementioned master's research: for the authors of this study, landscapes of investigation are fundamental for critical learning to take place in field education.

In order to introduce the reader to the theoretical basis, we will present a summary of the theoretical basis of the master's research from which this section originates, so that concepts of Field Education and Critical Mathematics Education can be elucidated. Next, we will present some contributions from the literature on working with Financial Education in the school context. We will detail the methodological paths taken to generate the data shown in this paper. We will then present the data produced from the development of the landscape of investigation and the analysis carried out. Finally, we will have a section with some partial conclusions from the master's research, which is still in progress.

2 Field Education

The idea of field education was born as a paradigm that opposed the paradigm of rural education. Fernandes and Molina (2004) point out that the origin of Field Education occurred with the *Encontro Nacional de Educadoras e Educadores da Reforma Agrária* (Enera — National Meeting of Educators of Agrarian Reform), held in July 1997. Caldart (2004) stresses that the *Conferência Nacional por uma Educação Básica do Campo* (National Conference for Basic Field Education), held in 1998, was the collective baptism of a new way of fighting and thinking about education for the Brazilian people who work and live in and from the rural areas. For the researcher, it was through the process of building this conference that a new reference for debate and popular mobilization was inaugurated: Field Education, and no longer rural education or education for rural areas.

The literature on field education highlights the need to think of it as a process of building an educational project based on the point of view of peasants and their trajectories of struggle. According to Caldart (2004), this means “thinking about education (politics and pedagogy) from the point of view of the social, political and cultural interests of a given social group” (p. 12).

Caldart (2004) considers that the field education can be one of the practical realizations of Paulo Freire's Pedagogy of the Oppressed, by affirming peasants as the legitimate subjects of an emancipatory and educational project. Consequently, from the researcher's point of view, one of the fundamental features of the identity of the movement *For a Field Education* is the struggle of rural people for public policies that guarantee their right to education, so that this

¹ This paper is part of a master's research project being carried out by the first author and supervised by the second author in the Postgraduate Program in Education at the School of Education, University of São Paulo.

education is in and of the countryside. “In: the people have the right to be educated in the place where they live; Of: the people have the right to an education designed from their place and with their participation, linked to their culture and their human and social needs” (Caldart, 2002, p. 18).

There is a conceptual, political and pedagogical difference between a *field school* and a *school in the countryside*. According to Silva (2019), a *field school* is one in which it is possible to identify work that is committed, contextualized and based on engagement between the school and the community. For the researcher, in addition to the school being part of the community, a *field school* requires the community to perceive the educational institution as an important reference. “The characteristics of the school and the community are intricate, strongly related, because in the school the community is identified and in the community the characteristics of the school are perceived” (Silva, 2019, p. 20).

On the other hand, according to the researcher, a *school in the countryside* does not carry these rural elements. For Silva (2019), “this is due to the absence of the teachers' sense of belonging to the school and the characteristics of the community, which is generally close to the city, among others” (p. 20).

We end this section by clarifying that the concept adopted for *field schools and schools in the countryside* follows the definition presented by Silva (2019). This will be important for characterizing the school where the landscape of investigation reported in this paper was carried out. In the following section, we will present characteristics of Critical Mathematics Education, a theory that is one of the most powerful trends in mathematics teaching and in the training of teachers who teach mathematics, as advocated by Milani and Marçal (2024).

3 Critical Mathematics Education

On the one hand, we have that “the reality that gave rise to this movement for a field education is one of violent dehumanization of living conditions in the countryside. A reality of injustice, inequality and oppression that demands urgent structural social transformations” (Caldart, 2002, p. 20). On the other hand, Skovsmose (2001) presents the most general and unifying idea for Critical Mathematics Education:

For education, both as practice and research, to be critical, it must discuss basic conditions for obtaining knowledge, it must be aware of social problems, inequalities, suppression, etc., and it must try to make education a progressively active social force (p. 101).

This is why, when delving into the theoretical field of Ole Skovsmose's Critical Mathematics Education, it becomes clear that its creator does not reduce it to a sub-area of Mathematics Education, nor to pedagogical methodologies and techniques or program content. For the researcher, “Critical Mathematics Education is the expression of concerns about Mathematics Education” (Skovsmose, 2014, p. 11).

Among these concerns, we can reflect on: the students' *background and foreground*; the dialog that can take place in the mathematics classroom; Mathematics in Action; teaching and learning conditions; the profound inequalities present in Brazilian education; as well as ways of engaging students in activities that break with traditional teaching methods and make them protagonists in the process of acquiring knowledge. Furthermore, it is possible to reflect on mathematics: “Mathematics refers not only to mathematical skills, but also to the competence to interpret and act in a social and political situation structured by mathematics” (Skovsmose,

2000, p. 68), among many other concerns.

Therefore, with the intention of offering an alternative to traditional teaching, which is full of endless lists of exercises, an alternative proposed by Skovsmose and other researchers in the field of Mathematics Education is investigative activities. These activities take place in a landscape of investigation, which, for Skovsmose (2000, p. 73), “is one that invites students to formulate questions and seek explanations”. Skovsmose (2014) also emphasizes that

a landscape of investigation is a field on which teaching and learning activities take place. Unlike the battery of exercises so characteristic of traditional math teaching, which presents itself as a safe and predictable road over the terrain, the trails of landscapes of investigation are not so well marked. There are different ways of exploring the terrain and its trails. There are times to proceed slowly and cautiously, and others to jump in wildly and see what happens (p. 45-46).

In CME theory, different types of references are possible in math activities. Firstly, it is possible to take into account that mathematical questions and activities can refer exclusively to mathematics. Secondly, it is possible to refer to a semi-reality, a reference that is not a reality actually observed, but a reality constructed, for example, by the author of a mathematics textbook. Finally, students and teachers can work with tasks that refer to real-life situations.

By combining the distinction between these three types of references and between the two paradigms of classroom practices (exercises and landscapes of investigation), Skovsmose (2000) obtains a matrix with six different types of learning environments, as can be seen in Table 1.

Table 1: Learning environments

	Exercises	Landscapes of Investigation
References to Pure Mathematics	(1)	(2)
References to semi-reality	(3)	(4)
References to reality	(5)	(6)

Source: Skovsmose (2000, p. 75)

After showing the matrix in Table 1 in his paper, Skovsmose (2000) states that it represents a simplification. The author points out that “the vertical line separating the exercise paradigm from the landscapes of investigation is certainly a very ‘thick’ line, symbolizing an immense terrain of possibilities” (Skovsmose, 2000, p. 81). This is because a math class based on Problem Solving, Game-Based Learning (or even Gamification), among others, already engages students to participate in the class in a way that causes a rupture in traditional teaching. That is, teaching based on lectures, in which the math teacher presents the content and some examples and then the students answer a *series* of exercises.

In his paper, Skovsmose (2000) points out that a large part of mathematics education alternates between environments (1) and (3). In this sense, for the author, the exercise paradigm offers a foundation based on the *tradition* of Mathematics Education. Thus, the researcher maintains that

Math Education should move between the different environments as presented in the matrix. In particular, I do not consider the idea of completely abandoning exercises in mathematics education. [...] It is important that students and teachers together find their way between the different learning environments (Skovsmose, 2000, p. 82).

Skovsmose (2014) clarifies that “we can invite, but never force, students to participate in activities around a landscape of investigation. Whether the invitation is accepted or not is always uncertain” (Skovsmose, 2014, p. 46). And that's why it's worth highlighting one more of the researcher's considerations:

Investigating and exploring are conscious acts; they don't happen as forced activities. They don't take place until the students actually carry out the investigations and explorations, and this presupposes that the students' intentionality is part of the investigative process (Skovsmose, 2014, p. 60).

So, on the one hand, we have the paradigm of field education breaking the paradigm of rural education. On the other hand, we have landscapes of investigation as an alternative path to the paradigm of exercise that is based on Mathematics Education and, in most cases, inhibits the possibility of criticality that can occur in the teaching and learning process in the Mathematics classroom.

So far, we have established some of the links between the field education and Critical Mathematics Education. Both theories aim to suspend certain practices that, with bad intentions or not, inhibit dialog between students and educators in the teaching and learning process, whether through the education of the rural area, which disqualifies the person from rural areas, or from a tradition rooted in the paradigm of the exercise in the math class, which does not provide an active participation of the student in their learning process.

Now, in order to build a bridge between Critical Mathematics Education and Financial Education, we will start with a concern present in Marco Aurélio Kistemann Júnior's doctoral thesis, which makes a relevant contribution to Financial Education. In his doctoral thesis, Kistemann Jr. (2011) points out that, in math classes, the teaching of Financial Education is often reduced to working with Financial Mathematics (in a simple and compound interest approach) and that “the teaching, when it does occur, of simple and compound interest, does very little to educate or enable the genesis of consumer individuals to deal with the economic daily life of the liquid-modern society” (Kistemann Jr., 2011, p. 190).

With this brief introduction to one of the concerns of Financial Education, we have established the bridge between theory and CME. For Skovsmose (2014, p. 12), “based on the idea of mathematics in action and the consequences of the use of mathematics in modern society, whether in economic, administrative, technological or all kinds of human activities”, we can conceive a critical view of mathematics.

Kistemann Jr. (2011) infers on the fruitful responsibility of Mathematics Education both in school environments and in the various social segments frequented by consumer individuals, understanding that if issues such as the use (or not) of Mathematics in society, the ideology of certainty and the Formatting Power of Mathematics are not problematized and questioned, we may be contributing to the maintenance of an asymmetrical and unjust society.

4 The Financial Education

By reading Kistemann Jr.'s doctoral thesis (2011), we understand that the aim of bringing Financial Education to students is not simply to present or promote access for individual-consumers to mechanical calculation rules for decision-making in their consumption practices. On the contrary, the aim is to promote analysis and reflection on consumption situations, even the most basic ones presented to the individual-consumer, fostering the critical participation of these individuals, through education that also focuses on the financial-economic

context.

The researcher envisions in Critical Mathematics Education

the perspective through which we can trace new methodological paths, involving Mathematics educators and teachers of Mathematics and other areas, drawing attention to the phenomenon of Financial Education and the process of empowerment that may be developing (Kistemann Jr., 2020, p. 49-50).

Thinking about the approach to Financial Education based on CME assumptions, some studies have sought to investigate Financial Education activities in textbooks from the perspective of Skovsmose's (2000) learning environments. Gaban and Dias (2014) developed this relationship in high school textbooks.

Reinforcing what Skovsmose (2000) already pointed out in his paper that brought the landscapes of investigation to a theoretical discussion in Brazilian Mathematics Education, Gaban and Dias (2014), explain that most of the Financial Education activities found in high school textbooks are in the type 3 learning environment (semi-reality in the exercise paradigm), which, according to the authors, is not desirable.

As a result of this observation, the researchers added to the project they developed “to suggest how such problems can be transformed into more interesting learning environments that are conducive to awakening in students an experience of mathematical reality, developing their criticality and consequently their social skills” (Gaban and Dias, 2014 p. 11).

In a way, the studies by Rosetti Jr. and Schimiguel (2011) corroborate what has already been said by Gaban and Dias (2016). These authors report having analyzed nine high school textbooks. As a result, they conclude that the focus of the books is on solving semi-real problems with direct application of the formulas presented, without discussion of the financial meanings related to them. This result is similar to that of the study by Barroso and Kistemann Jr. (2013).

Hermínio (2008) also presents the results of his analysis of nine high school textbooks. According to him, these textbooks distance the content studied at school from knowledge related to everyday life and the world of work, concluding that they practically do not address questions about Financial Mathematics in society.

Among the problems that permeate discussions about Financial Mathematics textbooks in Brazil, Queiroz and Barbosa (2016) highlight the concern with Financial Literacy; the distance between the content studied at school and knowledge related to everyday life and the world of work; the traditional way of approaching content; as well as references to technological resources and the way in which their use is encouraged, when it is (Barroso and Kistemann Jr, 2013; Caramori, 2009; Feijó, 2007; Hermínio, 2008; Rosetti Jr. and Schimiguel, 2011).

Having bridged the gap between Critical Mathematics Education and Financial Education, let's move on to some definitions in the field of Financial Education, in order to highlight what we intend to address throughout this text. According to Ferrari (2021),

Financial Education goes beyond financial mathematics, since the former is not just about calculation skills, but is also linked to emotions, habits and attitudes. Consequently, Financial Education becomes essential, since knowledge related to financial mathematics needs to be accompanied by people's critical view of everyday situations (Ferrari, 2021, p. 3-4).

In this sense, Kistemann Jr. (2020, p. 35-36) also points out that Financial Literacy can be promoted “not only as the acquisition of knowledge and content of Financial Mathematics, but by having the curricular content of this Mathematics to assist in decision-making in the daily economic life of consumers”.

To complement the idea of Kistemann Jr. (2020), we bring a consideration by Barbosa, Araújo and Paes (2020), who argue that Financial Education should occur from the Early Years of Basic Education, without being restricted to the teaching of Financial Mathematics and types of bank investments, but occurring from a critical perspective “that provides constructive reflections for students” (Barbosa, Araújo and Paes, 2020, p. 9).

Coutinho and Almouloud (2020) also make contributions to Financial Education. For the authors,

there are various definitions of financial education. However, many of them are concerned with banking aspects or even collective well-being, before even thinking about the formation of the individual as a person who is active and participative in their own financial present and future. This is not our interest, since we are dealing with school financial education and we understand financial education as the set of skills that lead the individual to manage their own financial life responsibly and with a view to their security and well-being. In the elementary school age group, it is important for young people to understand the value of money, to evaluate how this value varies over time and to manage it in order to make decisions that will improve their well-being (p. 77).

We conclude our theoretical presentation by considering that this approach has given the reader an understanding of the theoretical assumptions of Field Education, Critical Mathematics Education and Financial Education. Next, we will present a summary of the methodological paths we used to collect the research data, and then we will tell you how the landscape of investigation took place — the environment that stimulated critical learning for the students involved and made it possible to produce the data analyzed in the master's research of the first author of this paper.

5 The Methodological Path

As previously mentioned, this paper is a selection from a master's research project underway, classified as qualitative research in the field of Education. Bauer, Gaskell and Allum (2021, p. 23) point out that qualitative research “deals with interpretations of social realities”. The social reality analyzed in this master's research is that of the field education, which is present (or should be) in a school located in the rural area of a municipality in Bahia.

Lüdke and André (2022), in their book *Pesquisa em Educação: Abordagens Qualitativas* (Research in Education: Qualitative Approaches), point out that:

- Qualitative research has the natural environment as its direct source of data and the researcher as its main instrument;
- The data collected is predominantly descriptive;
- The concern with the process is much greater than with the product;
- Data analysis tends to follow an inductive process.

The reader will realize that the data presented in this paper is predominantly descriptive and that the analysis takes place through an inductive process, as pointed out by Lüdke and

André (2022).

In his master's research, the first author of this paper aims to understand how Critical Mathematics Education can contribute to the critical learning of students in the context of Field Education. In order to achieve this, two specific objectives were defined. Firstly, we turned to the scientific literature with the intention of analyzing the contributions of research and production in the area of Critical Mathematics Education to the field education. Then, going into the field, we sought to understand how projects based on landscapes of investigation can contribute to critical learning in the field education.

The field trip took place in a 3rd grade class in the Early Years of Elementary School at a municipal school located in the rural area of the city of Utinga, which is located in the Chapada Diamantina region in the state of Bahia. The aim of this field trip was to produce data so that the second specific objective of the master's research could be achieved. To this end, two landscapes of investigation were developed. Here, in this paper, we present one of these landscapes.

As this research involves human beings, the initial project was submitted to and approved by the Research Ethics Committee of the Faculty of Philosophy, Languages and Literature, and Human Sciences of the University of São Paulo, via Plataforma Brasil, and can be identified by the Certificate of Submission for Ethical Appraisal (CAAE): 71206723.0.0000.0138. To guarantee the anonymity of those involved, we chose to use fictitious names for the teacher and the students, which were inspired by the names of characters from the Harry Potter saga.

Based on what was shown in our theoretical discussion and the research carried out at the school, there are not enough elements to call it a *Field School*, but only a *school in the countryside*, as it is an institution located in a rural area that does not yet have a project that takes into account the specific characteristics of the rural people it serves. The Political-Pedagogical Project (PPP) is still under construction, and perhaps, with its implementation in the future, the school will begin to develop a Field Education Project.

The first research tool used in the field was participant observation. According to Denzin (1978 *apud* Lüdke and André, 2022), participant observation is a field strategy that simultaneously combines a number of research instruments. These include participation and direct observation, as well as introspection. We used participant observation. According to Oliveira (2022), when carrying out this technique, by using this technique, through the researcher's direct contact with the observed phenomenon, it is possible to obtain information about the reality of the social actors in their own context.

The participant observation mentioned here was part of participant research, the qualitative methodology used to fulfill the second specific objective. Oliveira (2022, p. 75) points out that, in participant research, “the involvement of researchers in the communities in which the study and research process takes place is of fundamental importance”. This involvement began, as already described, with participant observation and evolved into pedagogical interventions, i.e. the development of projects based on landscapes of investigation. It is important to emphasize that the use of participant research “stands out in education because the work itself is a teaching method, a way of educating” (Malheiros, 2011, p. 110).

In summary, the stages of the participant research we carried out were: participant observation; the development of the landscapes of investigation; the application of a questionnaire for the students to answer about their experiences of taking part in a master's

research project; and a round table discussion with all those involved in the study.

Having presented the theoretical and methodological assumptions of the master's research carried out by the first author of this paper, in the next section we will begin by presenting the research scenario, in order to make some considerations about the development of this learning environment.

6 Check out the fair!!! A Landscape of Investigation

The landscape of investigation described here was developed by simulating a trade fair. This reference was chosen for two reasons. Firstly, it was an activity that teacher Minerva had already been doing with her students in the 3rd year of Primary School. She said that in 2022, when she was working on the Brazilian currency with her students, she simulated a market and a clothes store, so that the students could exercise the mathematical calculations of the four operations in buying and selling situations. The second reason is related to the fact that working with marketing at the city's open market is part of the reality of two families of students in the class taking part in the research.

This landscape referred to a semi-reality, but had many elements that were similar to reality. It was a semi-reality because the students weren't at a real fair, but at a simulation, in which we had a greengrocer's, a grocery store, a clothes store and, to the students' surprise, a snack bar.

Students Dino, Gina, Harry and Hermione were in charge of sales at the greengrocer. Cho, Draco, Lilá and Luna took charge of sales at the grocery store. Cedrico, Hagrid, Neville and Rony were in charge of sales at the clothes store.

For the market, Minerva took some food from the school canteen, and Hagrid's grandmother provided some bottles of honey (which are produced in the settlement where Hagrid's family lives), to simulate sales, i.e. customers would buy these products, but at the end of the activity they would have to return them. The same logic applied to the clothes on display in the store. The students would simulate the purchases and, at the end of the activity, return the clothes, as Minerva had borrowed them from a store in the urban area of the city of Utinga. The fruit sold at the greengrocer's could be bought and eaten, as Minerva had asked all the pupils in the class to bring some fruit on the day of the fair. In addition, the teacher and the researcher also brought some more fruit.

The snack bar was a surprise. Teacher Minerva agreed with the first author of this paper about this stall, without telling the students. We invited fathers, mothers and guardians to be the traders in the snack bar. Only Hagrid's mother and Hermione's mother confirmed that they could help. However, on the day of the fair, Hermione's mother was unable to attend. So Hagrid's mother was in charge of the snack bar. The snack bar had the same logic as the greengrocer's: whatever was bought could be eaten, because what was sold there was produced by Professor Minerva: cakes, pies, brigadeiros, etc.

Professor Minerva distributed a certain amount of miniature money (notes and coins) so that those who would be acting as shopkeepers — the third year students and Hagrid's mother — could pass on the change. And we invited the 4th graders to be the customers. The first author of this paper was responsible for explaining to them how the dynamics of the fair simulation would work: the products bought at the clothes store and the grocery store would have to be returned at the end of the simulation, but the products bought at the greengrocer's and the snack bar could be consumed. The 4th graders were also given a certain amount of miniature money (notes and coins) to make their purchases. They were then taken to the 3rd

grade classroom and the fair simulation began.

When the fourth-graders entered the third-grade classroom, they immediately went to the snack bar to do their shopping. Some of them also went to the greengrocer's, but the grocery store and the clothing store *were empty*. The fact that they couldn't consume the products from these two stalls was probably the main reason why the 4th graders didn't seek them out at first. Then student Neville had an idea and started shouting *20 for 10, 20 for 10*. He was putting the clothes on sale. The 4th graders began to take an interest in the clothing store. Perhaps they had also eaten enough snacks and fruits to start browsing the other stalls.

At the greengrocer's, there were a lot of fruits that weren't being sold, such as oranges. We don't know the exact reason why the 4th graders weren't buying oranges, perhaps because they didn't have a knife to peel or split them. That's why they were prioritizing fruits like grapes, strawberries and plums, to be eaten right there. However, student Hermione had a strategy for selling the oranges. Inspired by the discounts Neville was giving in the clothes store, she also gave discounts on oranges. At the end of the fair, Hermione told us: *Nobody wanted to buy the oranges that were R\$ 3.00, so I changed it to R\$ 2.00 and everyone bought them.*

When the marketing moment was over, the 4th graders went back to their classrooms. The 3rd graders, on the other hand, had the task of checking the turnover of their sales. As they had sold a lot, we let them use calculators to calculate it.

At each stall, the group of students had a different strategy for calculating the turnover. The students in charge of the clothes store took each miniature note or coin and put its value into the calculator, adding it to the value of the next note or coin and so on. Because of this strategy, they miscalculated numerous times and had to restart the math several times, until teacher Minerva helped them with the stall's turnover, arriving at the figure of R\$ 3,944.00.

The students in charge of the grocery store started by organizing all the notes and coins into piles. One with R\$ 2.00 notes, another with R\$ 5.00 notes and so on. Then they counted how many notes they had in their respective piles. With the help of their teacher Minerva, they understood that they needed to multiply the value of the note by the number of notes in each pile to calculate the total amount of money in each of them. To conclude, they added up the value (in reais) of each pile and discovered that they had made R\$ 2,923.00.

The greengrocer's also organized the notes and coins into piles, but the process that followed was similar to the one used by the students in the clothes store. Hermione counted how many notes or coins they had in each pile and Harry added them up on the calculator value by value. In other words, when Hermione said they had eight R\$ 1.00 coins, Harry didn't add eight to the amount already in the calculator, he added $1 + 1 + \dots + 1$ eight times. And they did this process with the other coins and notes until they concluded that they had made R\$ 4,825.00.

The landscape of investigation ended when everyone, including Hagrid's mother, managed to calculate how much their stall had earned in that mock fair. After that, everyone went together to clean and organize the classroom and there was still some free time to play before the school day ended.

7 Landscape of Investigation Development Considerations

With this landscape of investigation, we were able to provide a lot of mathematical learning for the students by simulating a fair. This landscape of investigation made the students face some challenges with a lot of autonomy. For example, for Hermione, it was a great challenge to have several customers to serve and still make mental calculations quickly to give

change. At one point, she had a pencil and paper in her hand to calculate how much change she had to give to one of her customers.

As well as facing some challenges, taking part in a trade fair simulation gave the students the opportunity to exercise/develop their mental calculation skills. At the end of all the marketing, they had plenty of autonomy to choose which method to use in order to calculate their sales revenue. Once again, we broke away from the practices of the exercise paradigm, in which the teacher usually tells the students how to do things. They made many mistakes, but in the end they achieved their goals by accepting the invitation to take part in the landscape of investigation.

In addition to learning mathematics, the students put into action experiences that they see their family members experiencing, or even when they go to the city market. An example of this is the strategy of offering discounts, which was present in two stalls and came up as an idea from the students themselves during the trade fair simulation.

To conclude our participant research, in the last two stages, the students were asked about their participation in the investigative activities. Initially, they answered a questionnaire and then took part in a round table discussion in which we reflected on our experiences in the investigative activities.

The questions that referred to the investigative activity reported in this paper were as follows:

- In your opinion, how was mathematics present in the activity involving the buying and selling of products (clothes store, grocery store, greengrocer)?
- Tell us a little about your experience investigating the activity involving buying and selling products (clothes store, grocery store, greengrocer)?

The students answered these questions and, as we analyzed their answers, we highlighted some to share with everyone during the round table discussion. For the first question, the following answers were highlighted:

By giving a discount and to sell we gave change. And we calculate how much we earn from sales. (Rony)

To count money when I sold and to calculate how much I sold in total. (Cho)

In the account to find out how much we had earned and in the change to give to others. (Harry)

As for the second question, the following answers stood out:

Some products took longer to sell. (Neville)

I learned how to give change and realized that when we offer a discount, people are interested. (Hermione).

In the little shop we sold a lot and made a lot of money. If only it were real! (Harry)

The focus of the first question was to identify the students' perception of how school mathematics was present in the landscape of investigation, which referred to an activity that two of the students' families in that class practiced for family subsistence. With the answers, it was possible to identify that the students were able to get an idea of how the relatives of some of the students in the class need to use Mathematics in Action when they make sales at the town fair: giving change; checking how much they made at the fair; giving a discount; counting

money, etc.

When we asked the second question, our intention was to highlight what the students perceived, in general, when they experienced an educational practice that distanced itself from traditional teaching based on the exercise paradigm. This question and its answers were essential for us to understand how projects based on landscape of investigation can contribute to critical learning in field education, which is one of the specific objectives of the master's research conducted by the first author of this paper.

The questions and some of the answers, such as those highlighted above, were presented to the students during the round table discussion. At this point, they viewed their colleagues' perceptions of Math in Action, in an activity based on marketing, and identified commonalities between their answers.

At some points during the round table discussion, the students shared stories about what happened at their stalls during the fair. Hagrid and Rony mentioned that a customer gave them a R\$ 200 bill to buy a R\$ 20 piece of clothing. However, as they were taking too long to calculate the amount of change, the customer left without taking it. Student Neville, who was also *working* in the clothes store, intervened in the conversation, saying that they should have given the customer R\$ 100.00 in change, but he didn't explain how they arrived at that amount.

Here, we see that, of the three shopkeepers in the clothes store, Hagrid and Rony were concerned with performing the mathematical calculations correctly when giving their customers change, even if it took them a while to do the calculations mentally. The student Neville, on the other hand, would *guess* any number for the result of the change or make a mistake in the calculation, because his intention was to resolve the negotiation quickly.

The term *guess* was used in the previous paragraph because, during the math classes that were part of the participant observation, Neville used to guess answers when we asked him the result of a calculation. Teacher Minerva even drew his attention to this practice several times. Neville, you're saying any number, do the math properly to give the answer.

In relation to this episode shared by the people in charge of the clothes store, Hagrid and Rony reported that, together, they managed to calculate that the change was R\$ 180.00. The customer returned to the store and they gave him the correct change. Student Harry took part in this conversation and said: *So it's all right, not much has changed. You've only earned an extra R\$ 20.00.*

Hermione then recounted an experience of selling at the grocery: *The customer gave me R\$ 10.00 worth of something there, which I can't remember, it was R\$ 2.00, so she had to help me.* During the fair, the first author of this paper walked around the stalls, watching the students sell and buy, taking some photos and making some videos. This was exactly the marketing experience that Hermione wanted to share with her colleagues. We took advantage of Hermione's story to ask: *What change did Hermione have to give her customer?* Student Harry replied: *Four R\$ 2.00 notes.* Then he was asked: *So how much is four R\$ 2.00 notes?* And again student Harry replied: *Eight.*

The moment when Neville was putting the clothes on sale was also filmed. The video was played and everyone watched the video of Neville shouting *20 for 10, 20 for 10* to the whole class. And we asked: *What is the intention when we reduce the price like this at the fair?* Student Harry said it was to run a promotion *to make more money.* Then a new question arose: *Why make more money?* Hermione contributed by saying: *People will want to buy it because it's cheap.* Professor Minerva also took part in this conversation and asked the students: *And*

the lower the price of the goods, the more...? Student Rony said: *...the more it sells*. Student Neville said: *...the more you earn*, which means that his intention in giving discounts was to attract customers in order to earn more money.

The students discussed some controversial practices that took place during the fair. Neville said: *“Oh, teacher, that's unfair, I went to buy a mango and Harry said, “no, no, it's already sold”. But there were two. Hermione confirmed: It was already sold. Harry said: It was already sold, the customer was waiting. Professor Minerva said to Neville: Neville, that means you didn't arrive first. Whoever got there first took it. And Hermione added: The customer said, ‘Keep it here and I'll come and pick it up later’*. So everyone managed to explain to Neville why they hadn't sold him the mango.

During the round table discussion, the students had the opportunity to share their experiences of taking part in the landscape of investigation. They expressed that there were times when they were nervous and thought they *wouldn't be able to cope*. They were very nervous when they had to be quick with their mental calculations, because customers don't wait for long during a sale. However, working together as a team (Hagrid and Rony) and being helped by colleagues from another class (Hermione), they faced their nervousness/fear and played a good role during an investigative activity, something that is very similar to Mathematics in Action.

In fact, moving from the exercise paradigm to landscapes of investigation puts everyone in a risk zone, as observed by Penteado (2001). However, by experiencing these risks and engaging with great autonomy and criticality in an activity with reference to Financial Education, the students had the opportunity to establish links between school Mathematics and Mathematics in Action, present in an activity that family members of rural students carry out for the subsistence of their families. This approach is desirable in a math class in the context of field education.

8 Final considerations

Our understanding of how landscapes of investigation can contribute to critical learning in field education is in line with the contributions of our patron of education, Professor Paulo Freire. Through our participant research, we understand that, in the context of field education, landscapes of investigation in mathematics class provide the exercise of citizenship by democratizing the possibility of the naturalness of mathematics as a condition of being in the world.

This naturalness of mathematics occurred in the aforementioned participant research when the landscapes of investigation provided the students with mathematical learning and learning beyond mathematics. In other words, learning that comes from the articulation of school mathematics with the reality of the countryside of the students taking part in the research, in other words, with the development of mathematical skills.

When students are provided with mathematical skills, they develop the competence to interpret and act in a social and political situation structured by mathematics. In the case of this research, we provided the students with a landscape of investigation with reference to semi-reality, which was linked to Financial Education, as it took place through the simulation of a trading activity.

Acknowledgment

This paper was carried out with the support of the Coordenação de Aperfeiçoamento de Pessoal

de Nível Superior (CAPES — Coordination for the Improvement of Higher Education Personnel — Brazil) — Funding Code 001.

Note

The translation of this paper from Portuguese into English was funded by the Minas Gerais State Research Foundation (Fapemig — *Fundação de Amparo à Pesquisa do Estado de Minas Gerais*), under Call for Proposals 8/2023.

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