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# Algebraic thinking in Elementary School: the perception of regularities and relational thinking

***Abstract:*** With this study, we proposed to map and discuss the Brazilian academic research that has as its object of study the development of Algebraic Thinking in Elementary School in the interstice from 2017 to 2023. The methodological approach employed is qualitative, of the mapping typology. The results reveal a growing body of research involving algebraic thinking in the context of the initial and continuing training of teachers who will teach and who teach mathematics in Elementary School, so further analysis of future research is needed to try to identify these realities, perhaps looking at the curricular components studied in the initial training courses and the profile of this teacher who will arrive at the school and also the courses offered to teachers who have already joined a school unit.

***Keywords:*** Algebraic Thinking. Elementary School. Mapping.

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## Pensamiento algebraico en la Primaria: una comprensión de las producciones académicas brasileñas

***Resumen:*** Con el estudio, nos propusimos mapear y discutir la investigación académica brasileña cuyo objeto de estudio fue el desarrollo del Pensamiento Algebraico en la Primaria, en el período de 2017 a 2023. El enfoque metodológico utilizado es cualitativo, de tipología de mapeo. Los resultados revelan una creciente investigación que involucra el Pensamiento Algebraico en el contexto de la formación inicial y continua de los docentes que enseñarán y enseñan Matemáticas en la Primaria, requiriendo nuevos análisis de futuras investigaciones para tratar de identificar estas realidades, tal vez considerando los componentes curriculares estudiados en los cursos de formación inicial y el perfil de este docente que llegará a la escuela; También se analizan los cursos ofertados a los docentes que ya se han incorporado a una unidad escolar.

***Palabras clave:*** Pensamiento Algebraico. Primaria. Mapeo.

## Pensamento algébrico nos Anos Iniciais do Ensino Fundamental: uma compreensão sobre as produções acadêmicas brasileiras

***Resumo:*** Com o estudo propusemos mapear e discutir as pesquisas acadêmicas brasileiras cujo objeto de estudo foi o desenvolvimento do Pensamento Algébrico nos Anos Iniciais do Ensino Fundamental, no interstício de 2017 a 2023. A abordagem metodológica empregada é qualitativa, de tipologia mapeamento. Os resultados revelam uma crescente pesquisa envolvendo o Pensamento Algébrico no contexto da formação inicial e continuada dos professores que ensinarão e que ensinam Matemática nos Anos Iniciais, cabendo novas análises de pesquisas futuras para tentar identificar essas realidades, talvez considerando os componentes curriculares estudados nos cursos de formação inicial e o perfil deste professor que chegará à escola; também analisando os cursos ofertados para os professores que já ingressaram em alguma unidade escolar.

***Palavras-chave:*** Pensamento Algébrico. Anos Iniciais. Mapeamento.

## 1 Introduction

The National Council of Teachers of Mathematics — NCTM (2007) recognized the

importance of algebra as a cross-cutting theme, given that it cuts across the other axes that make up the mathematics curriculum. This document states that students should be able to understand patterns, functions and relationships, as well as "analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; analyze variation in various contexts" (p. 39).

With regard to the first years of schooling, the NCTM (2007) recommends that students should be able to experiment and develop their knowledge and gradually take ownership of algebraic representations and symbologies, from the perspective of Generalized Arithmetic or Relational Thinking, which according to Mescouto, Lucena and Barbosa (2021) consists of looking at numbers beyond the exclusive focus on calculation procedures, as well as "focusing on their understanding and the use of a set of existing relationships, as well as the properties of operations and the equal sign" (p. 5).

With regard to the conceptualization of generalization, authors such as Ponte, Branco and Matos (2009) have defended it as an idea that has proved to be a promising path for the development of algebraic thinking. According to Ponte, Branco and Matos (2009, p. 10), generalization is

> "an element that is also central to algebraic thinking is the idea of generalization: discovering and proving properties that are verified in a whole class of objects", which can be conceived as the core of algebraic thinking and is also part of mathematical reasoning.

Ponte, Branco and Matos (2009) state that the purpose of studying algebra in Elementary and Middle School is to develop algebraic thinking in students, which is characterized as the ability to deal with algebraic expressions, equations, inequalities, systems of equations and inequalities and functions, but also includes "the ability to deal with other mathematical relations and structures and use them in the interpretation and resolution of mathematical problems or other domains" (p. 10). In this sense, algebraic thinking focuses not only on objects, but also on the relationships between them, representing and reasoning about these relationships in a general and abstract way. The authors go on to say that one of the best ways to encourage this thinking is to study regularities in a given set of objects.

Understanding Algebra and Algebraic Thinking shows that the approach is not reduced to formal symbolism; in other words, learning Algebra involves being able to think algebraically in a variety of situations, including relationships, regularities, variation and modeling. Reducing algebraic activity to symbolic manipulation means limiting the richness of algebra to just one of its facets (Ponte, Branco and Matos, 2009).

The Base Nacional Comum Curricular [National Common Curricular Base — BNCC] (Brasil, 2017) emphasizes the development of algebraic thinking in Elementary School, stressing that it is essential for understanding mathematical relationships and patterns, exposing students to situations that involve the identification and generalization of regularities, promoting the recognition of patterns, which are an essential part of the development of algebraic thinking. development of this type of thinking, encouraging the formulation of conjectures for this mathematical learning. This development gives us a profound construction of mathematical concepts, preparing students for future challenges in mathematical learning, facilitating this transposition from arithmetic to algebraic thinking.

FIn view of these considerations, this study is the result of ongoing Master's research in the Postgraduate Program in Science Teaching at the Cruzeiro do Sul University and is located

in the Curriculum, Assessment and Teacher Training in Science and Mathematics Teaching Research Line within the scope of the Research Project entitled *Contributions of research to the teaching and learning process in Mathematics in the 5th year of Elementary School: a pilot project involving two Teaching Directorates of the São Paulo State Network,* coordinated by the second author of this text and with funding from the National Council for Scientific and Technological Development (CNPq).

Thus, in this text, we take as our starting point the guiding question: *What does Brazilian academic research, in the period from 2017 to 2023, reveal about the construction and development of Algebraic Thinking in Elementary School?* Based on this central question, we propose to map and discuss Brazilian academic research whose object of study is the development of Algebraic Thinking in Elementary School, in the period from 2017 to 2023, with a view to understanding the processes underlying the development of Algebraic Thinking in the early stages of schooling. To this end, we opted for a qualitative, mapping approach.

In the next section, we present a discussion of the concept of algebraic thinking. We then discuss the methodological option. Afterwards, we explain our discussions and finally, we make some considerations about the data collected.

## 2 Algebraic Thinking

In this section, we focus our discussions on the importance of Algebraic Thinking in the light of some theorists who discuss the nuances of such thinking in Elementary School. First, however, we turn our gaze to the National Common Curricular Base — BNCC (Brasil, 2017) in order to analyze, according to what this reference document presents about Algebraic Thinking, emphasizing how this development is carried out and how it is foreseen for Elementary School, the focus of this text.

In the BNCC (Brasil, 2017), Algebraic Thinking is linked to the Thematic Unit Algebra, as a special type of thinking that will be developed essentially to use mathematical models to understand, represent and analyze quantitative relationships between quantities.

The BNCC (Brasil, 2017) emphasizes the importance of Algebraic Thinking from the Elementary School and proposes that this thinking be incorporated into everyday school life gradually, starting with the recognition of patterns, relationships between numbers and sequences, thus creating a solid foundation for the more advanced concepts of Algebra to flow better. This is followed by the use of representations and modeling, since the document also encourages their use, and different representations so that they can understand algebraic concepts in the light of their own strategy.

Another aspect of the document that is linked to Algebraic Thinking concerns the Problem Solving, recognized as the Mathematical Process. In the BNCC (Brasil, 2017), it is fundamental for students to realize the relevance of such thinking in their daily lives, often breaking the barrier of distance between algebra and its applicability.

Algebraic thinking has been discussed in the light of various theorists. Kaput (2008) points out that algebraic thinking is not just a preparation for formal algebra, but an essential skill that should be developed from the earliest school years.

Kaput (1999) argues that algebraic thinking should be seen as a natural extension of arithmetic skills, highlighting the importance of building conceptual bridges between arithmetic and algebra. From this perspective, Ponte (2010) emphasizes the importance of integrating algebraic thinking into the curriculum from the earliest years of schooling. This integration

becomes fundamental for the development of mathematical relationships and their structures.

We recognize that both authors agree that the transition from arithmetic to algebra should not be abrupt, but gradual and supported by teaching that values symbolic reasoning and the generalization of patterns.

While Kaput (1999) emphasizes that the introduction of algebraic thinking should be anchored in students' existing arithmetic knowledge, Ponte (2010) adds that this introduction should promote an articulation between different mathematical representations, such as verbal, graphic and symbolic.

Ponte (2010) argues that the teaching of algebra should go beyond the manipulation of symbols, encouraging students to reflect on the meaning and applicability of algebraic operations in different contexts. This perspective complements Kaput's (1999) view, suggesting that the creation of meanings and the connection with students' prior knowledge are fundamental for meaningful algebra learning.

Kaput (1999) warns of the challenges of teaching algebra in such a way that students really understand the concepts behind the symbols. Ponte (2010) corroborates this view by emphasizing that teaching algebraic thinking should promote critical reflection on mathematical ideas, rather than focusing exclusively on manipulating formulas and equations.

We understand that, for both authors, the key to successful algebra teaching lies in an approach that encourages students to see algebra as a natural continuation of what they already know, but which also challenges them to think abstractly and construct new mathematical meanings.

Kaput (2008) proposes that algebraic thinking should be approached in a continuous and progressive way, a view that complements Ponte's (2010) idea that generalization and pattern recognition are fundamental to mathematical development.

By interrelating the perspectives of Kaput (1999) and Ponte (2012), it is clear that both authors agree on the importance of algebraic thinking as a transversal and fundamental skill for Mathematics Education. Kaput (1999) emphasizes the expansive nature of algebraic thinking, while Ponte (2010) reinforces the need to cultivate these skills from the earliest school years, highlighting the importance of teaching that goes beyond mere symbolic manipulation to include generalization and reasoning. This combination of ideas provides a rich and multidimensional view of algebraic thinking, which is essential for students' mathematical education.

By introducing Algebraic Thinking from the earliest years, educators provide students with a solid foundation that prepares them for more complex challenges in the future. This approach promotes the ability to identify regularities and formulate conjectures, which not only facilitates the transition to formal algebra, but also enriches mathematical understanding in various other areas. Therefore, Algebraic Thinking acts as a thread that connects different aspects of Mathematics, allowing students to develop an integrated view of knowledge.

Furthermore, fostering Algebraic Thinking in children contributes to a more positive attitude towards Mathematics. Activities that involve exploration and discovery make learning more attractive and meaningful. When students feel motivated and engaged, they not only improve their problem-solving skills, but also cultivate a lasting interest in the subject. Thus, as Kaput (2008) argues, it is crucial that algebraic thinking is a central focus from the early school years, laying a strong foundation for future learning.

According to Ponte, Branco and Matos (2009), we can identify three essential strands of Algebraic Thinking. Firstly, there is the Representation strand, which involves the ability to use different symbolic representation systems. Secondly, there is the aspect of Reasoning, which encompasses both deduction and induction, allowing the analysis of mathematical properties, which involves relating and generalizing, establishing connections between specific classes of objects. Finally, we find the strand of *Solving problems and modeling situations*, which refers to the ability to create models for various situations and use different representations to interpret and solve problems in mathematics and other fields. Table 1 summarizes these fundamental aspects of algebraic thinking.

Table 1: Fundamental aspects of algebraic thinking

| **Strand** | **Strand Description** |
|---|---|
| Represent | Read, understand, write and operate with symbols using the usual algebraic conventions;<br>Translate information represented symbolically into other forms of representation (e.g. verbal, numerical, tables, graphs) and vice versa;<br>Demonstrate a sense of symbol, namely by interpreting the different meanings of the same symbol in different contexts. |
| Reasoning | Relate (in particular, analyze properties);<br>Generalize and act on these generalizations, showing an understanding of the rules;<br>Deducing. |
| Solving problems and modeling situations | Use algebraic expressions, equations, inequalities, systems (of equations and inequalities), functions and graphs to interpret and solve mathematical problems and problems in other areas (modeling). |

Source: Ponte, Branco and Matos (2009)

The development of algebraic thinking in Elementary School is essential for building a solid foundation for understanding advanced mathematics. As Ponte (2009) points out, this type of reasoning facilitates the transition from arithmetic to algebraic thinking, promoting a deeper understanding of mathematical relationships and patterns. By introducing algebraic concepts at an early age, students become able to identify patterns, formulate generalizations and solve problems in a more effective way. effective. This approach not only enriches students' mathematical understanding, but also strengthens their cognitive and analytical skills, which are fundamental for learning in other disciplines.

In this sense, Ponte (2009) makes an important point about the perspective of algebraic thinking and its aspects, which also includes an ability to deal with some other mathematical relations and structures to the point of using them in the interpretation and resolution of problems of a mathematical order or other domains, that is, the development of this thinking can be used in the educational sphere in order to provide the student with subsidies for an analytical understanding of subjective situations of that reality.

## 3 Methodology

The study in question adopts a qualitative approach, of the Mapping typology, with the aim of presenting some considerations of the knowledge produced in the field of study. We intend to produce a portrait of research on the theme of Algebraic Thinking, in the defined period, in order to indicate trends and understandings from this research in the scope of theses

and dissertations from the database of theses and dissertations of the Coordination for the Improvement of Higher Education Personnel (CAPES).

The typology mapping methodology, as a systematic process, seeks to

> a survey and description of information about the research produced on a specific field of study, covering a given space (place) and period of time. This information concerns the physical aspects of this production (describing where, when and how many studies were produced over the period and who were the authors and participants in this production), as well as its theoretical-methodological and thematic aspects (Fiorentini *et al*., p. 18).

In an attempt to guide and organize the mapping, we were inspired by Possamai and Allevato (2022) who follow a scheme of the whole process in the light of Ramos, Faria and Faria (2014), which although it is used for literature review, we envisage if possible for research of the Mapping Type.

Table 2: Mapping Protocol

| | |
|---|---|
| Study problem | What does Brazilian academic research from 2017 to 2023 reveal about the construction and development of algebraic thinking in Elementary School? |
| Research equations | We used the search expression: algebraic thinking. |
| Scope of research | Bank of Theses and Dissertations of the Coordination for the Improvement of Higher Education Personnel (CAPES). |
| Inclusion criteria | We selected research from 2017 to 2023, from postgraduate programs in Education, Mathematics Education and Mathematics Teaching. |
| Exclusion criteria | We excluded research that was outside the intended thematic scope and that did not include at least one theoretical framework related to Algebraic Thinking. |
| Data processing | The data was analyzed based on categories of readings organized into folders named by year and thematic focus: initial training, continuing training and classroom practices. |

Source: Possamai and Allevato (2022) adapted from Ramos, Faria and Faria (2014)

In view of the above and challenged to answer the guiding question, we began our process by identifying academic productions in the Bank of Theses and Dissertations of the Coordination for the Improvement of Higher Education Personnel (CAPES). This reference database gathers dissertations and theses defended in Stricto Sensu postgraduate programs since 2017.

In order to delimit the research that would form part of the research corpus, we decided to start the search by listing the descriptor Algebraic Thinking. From this search, we found 213 studies, some of which were far from the real objective of the study. Therefore, the results of this search were narrowed down by inserting the typology Master's, Professional Master's and Doctorate as a filter, thus arriving at a number of 192 studies to be analyzed. However, we still found some studies that were far from what was at the heart of the research, so we added another filter, period, and selected studies from 2017 to 2023, reaching a total of 122 studies. We justified the choice of this period because Algebraic Thinking in Elementary School was

officially introduced following the promulgation of the BNCC (Brasil, 2017).

Table 3 shows the number of studies carried out during this first survey.

Table 3: List of Theses and Dissertations referring to the descriptor *Algebraic Thinking*

| Type | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | Total |
|---|---|---|---|---|---|---|---|---|
| Dissertations (Academic) | 5 | 4 | 9 | 9 | 10 | 15 | 10 | 62 |
| Dissertations (Professional) | 2 | 2 | 10 | 3 | 10 | 5 | 9 | 41 |
| Theses | 1 | 1 | 1 | 3 | 2 | 4 | 7 | 19 |

Source: Research data

The focus of the research is on Algebraic Thinking in Elementary School, which is why we added another filter, Elementary School, which reduced the number of studies to a very small number, 43 from 2017 to 2023, as shown in Table 4.

Table 4: List of Theses and Dissertations referring to the descriptors *Algebraic Thinking and Elementary School*

| Type | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | Total |
|---|---|---|---|---|---|---|---|---|
| Dissertations (Academic) | 2 | 0 | 5 | 4 | 5 | 8 | 3 | 27 |
| Dissertations (Professional) | 0 | 0 | 2 | 3 | 1 | 1 | 0 | 7 |
| Theses | 0 | 0 | 0 | 2 | 0 | 3 | 4 | 9 |

Source: Research data

We then found ourselves with this impasse, observing and analyzing a large number of studies that might not corroborate our concerns as researchers, so we decided to align the studies starting with the abstracts. Right from the start, we found out that several studies, even if they dealt with Algebraic Thinking in Mathematics Education Debate, Montes Claros, v. 8, n. 14, p. 1-26, 2024 their title, did not include this in their abstract or in the research report, and did not even discuss the defined theme. We therefore ended up with 33 papers to make up our corpus of analysis, as shown in Table 5.

Table 5: Research corpus

| Year | Type of the Research | Title | Authorship and Guidance |
|---|---|---|---|
| 2017 | Dissertation | Algebraic Thinking in Elementary School: the perception of regularities and relational thinking | Author: Carla Cristiane Silva Santos<br>Advisor: Daniela Dias dos Anjos |
| 2017 | Dissertation | Algebra in Elementary School: an analysis of mathematical knowledge about Algebraic Thinking | Author: Mirian Criez Nobrega Ferreira<br>Advisor: Alessandro Jacques Ribeiro |
| 2019 | Dissertation | The development of algebraic thinking mediated by digital technologies in Elementary School | Author: Francislaine Alves de Souza<br>Advisor: Francine de Paulo Martins Lima |
| 2019 | Dissertation | Exploratory-investigative tasks for the development of algebraic thinking in Elementary School: an experience for thinking about the teaching-learning-evaluation relationship | Author: Juliana Batista de Sousa<br>Advisor: Isabel Cristina Rodrigues de Lucena |
| 2019 | Dissertation | Written mathematical communication | Author: Jane Lopes de Sousa Goma |

| | | | |
|---|---|---|---|
| | | involving algebraic thinking with future Elementary School teachers | Advisor: Ana Lucia Manrique |
| 2019 | Dissertation | Elementary School teachers' knowledge and the equal sign: an investigation with professional learning tasks | Author: Lilian Cristina de Souza Barboza<br>Advisor: Alessandro Jacques Ribeiro |
| 2019 | Dissertation | Algebraic Thinking from the perspective of Objectification Theory: an analysis based on episodes of joint work in the 5th grade of Elementary School | Author: Jessica Goulart Da Silva<br>Advisor: Ricardo Fajardo |
| 2019 | Dissertation | A study on the relationship between the development of algebraic thinking, self-efficacy beliefs, attitudes and specialized knowledge of pre-service and in-service teachers | Author: Roseli Regina Fernandes Santana<br>Advisor: Nelson Antonio Pirola |
| 2020 | Dissertation | Introducing Algebraic Thinking in Elementary School | Author: Eva Lucia Teodoro<br>Advisor: Saddo Ag Almouloud |
| 2020 | Dissertation | Algebraic Thinking: an exploratory study with Pedagogy students | Author: Rosilda do Santos Nascimento<br>Advisor: Maria Alves de Azeredo |
| 2020 | Dissertation | Algebraic Thinking in Elementary School: an application proposal | Author: Vagner Campeão<br>Advisor: Tulio Oliveira de Carvalho |
| 2020 | Dissertation | Algebra in Elementary School Mathematics curriculum: what now? | Author: Adriana Jungbluth<br>Advisor: Everaldo Silveira |
| 2020 | Theses | Introduction to Algebra in Elementary School: an analysis based on Objectivation Theory | Author: Luanna Priscila da Silva Gomes<br>Advisor: Claudianny Amorim Noronha |
| 2020 | Dissertation | Development of algebraic thinking among Elementary School teachers in teaching activities: theoretical thinking mediated by algebraic concepts | Author: Fernanda Cristina Ferreira Dos Santos<br>Advisor: Vanessa Dias Moretti |
| 2020 | Theses | Investigation into/of one's own practice: the intertwining of the development of algebraic thinking of students in the first year of Elementary School with the processes of teacher self-education | Author: Katia Gabriela Moreira<br>Advisor: Adair Mendes Nacarato |
| 2020 | Dissertation | Teaching Algebra in Elementary School: a proposal for a continuing education course in the light of the BNCC's ideas | Author: Taina Lopes da Silva<br>Advisor: Aline Caetano da Silva Bernardes |
| 2021 | Dissertation | Pedagogical knowledge related to the development of Algebraic Thinking in Elementary School: analysis of scientific productions in the area, official documents and education | Author: Debora Cristine Frasnelli<br>Advisor: João Bernardes Da Rocha Filho |

| | | | |
|---|---|---|---|
| | | evaluation systems in Brazil | |
| 2021 | Dissertation | Alice in the country of collaboration: algebraic thinking in Elementary School | Author: Renata Cristine Conceição<br>Advisor: Regina Celia Grando |
| 2021 | Dissertation | Algebraic Thinking in Elementary School: a look at scientific research that contributes to learning what is proposed by the BNCC | Author: Raquel Guimaraes de Medeiros<br>Advisor: Gabriela dos Santos Barbosa |
| 2022 | Dissertation | The enigma of the pussycat: connecting literature with algebraic thinking (stimulating literature for the development of algebraic thinking) | Author: Priscila Tereza Rodrigues Lanes Souza<br>Advisor: Rafael Montoito Teixeira |
| 2022 | Dissertation | Algebraic Thinking from the perspective of Elkonin Davydov's developmental teaching: a study with Elementary School teachers | Author: Nataliya Malska<br>Advisor: Circe Mary Silva da Silva Dinnikov |
| 2022 | Dissertation | Algebraic thinking in the 5th year of middle school: exploring missing value tasks | Author: Anailde Felix Marques<br>Advisor: Jadilson Ramos de Almeida |
| 2022 | Theses | Algebraic Thinking in Elementary School: a look at teacher expressions | Author: Vanessa de Oliveira<br>Advisor: Rosa Monteiro Paulo |
| 2022 | Dissertation | (Re)Seeing the continuing education of teachers who teach mathematics when it comes to algebraic thinking: limits and challenges | Author: Danielle Abreu Silva<br>Advisor: Carmen Lucia Brancaglion Passos |
| 2022 | Dissertation | Evidence of Elementary School teachers learning about algebraic thinking in a study group | Author: Jocelei Miranda da Silva<br>Advisor: Klinger Teodoro Ciríaco |
| 2022 | Dissertation | The introduction of Algebra in Elementary School with an emphasis on Algebraic Thinking: an analysis from the perspective of the Anthropological Theory of the Didactic | Author: Reinaldo Manoel da Silva<br>Advisor: Edelweis Jose Tavares Barbosa |
| 2022 | Dissertation | Algebraic Thinking in Elementary School in academic works produced in Brazil over the last three decades | Author: Tchierly Juliani Bier de Oliveira<br>Advisor: Dulcyene Maria Ribeiro |
| 2022 | Dissertation | Continuing education: a new look at the teaching of Patterns, Regularities and Sequences in Elementary School | Author: Mirian Cristina Bleich Muller<br>Advisor: Viviane Clotilde da Silva |
| 2022 | Dissertation | Continuing education for Elementary School teachers in the remote context: a look at objectification processes in pattern generalization tasks | Author: Zaine Hete Ribeiro De Oliveira<br>Advisor: Jadilson Ramos de Almeida |
| 2022 | Theses | Investigating how and what knowledge teachers should mobilize in their practices in order to promote | Author: Anderson Adelmo da Silva<br>Advisor: Barbara Lutaif Bianchini |

| | | | |
|---|---|---|---|
| | | the development of Algebraic Thinking in Elementary School | |
| 2023 | Theses | Building professional knowledge for teaching algebraic thinking in Elementary School: learning in a community of practice | Author: Vera Cristina de Quadros<br>Advisor: Susana Paula Graça Carreira |
| 2023 | Theses | Forms of generalization in the teacher training process involving elements of algebraic knowledge in Elementary School | Author: Iraji de Oliveira Romeiro<br>Advisor: Vanessa Dias Moretti |
| 2023 | Theses | From a continuing education process to teaching practice: the paths of teachers' professional learning about algebraic thinking in Elementary School | Author: Daniela Ines Baldan da Silva<br>Advisor: Alessandro Jacques Ribeiro |

Source: Research data

Based on this corpus, we looked at these 33 studies to analyze what is being discussed about Algebraic Thinking within this chosen timeframe. We know that, faced with so many selected studies, it was necessary to work on the priorities of our analysis, and in view of this, we delimited some important aspects so that the analysis would not be so far removed from the reality of the studies themselves.

Having delimited these studies, we first began to analyze the reality in which they were inserted, each one in its own particular way. We then realized that the discussion of Algebraic Thinking goes far beyond the classroom, and can take shape in the discussions from the reality of teachers in their daily practice to debates about the knowledge of these teachers in the reality of their initial training.

It is not our intention to compile a compendium of all the subjects dealt with in these research objects, but rather to look for indications of what these studies are talking about and how they relate to the topic of Algebraic Thinking in Elementary School.

Before moving on to our discussions, we would like to give an overview of the distribution of the corpus of analysis. Firstly, we present the number of studies by type of research and year.

Table 6: Distribution of research by Type and Year

| Year | Dissertation | Theses |
|---|---|---|
| 2017 | 2 | 0 |
| 2018 | 0 | 0 |
| 2019 | 6 | 0 |
| 2020 | 6 | 2 |
| 2021 | 3 | 0 |
| 2022 | 9 | 2 |
| 2023 | 0 | 3 |
| Total | 26 | 7 |

Source: Research data

As we can see in Table 5, there are 33 studies that are in line with our objective. The majority (26) are academic and professional Master's dissertations. We hypothesize that the reduced number of theses was due to the fact that algebraic thinking in Elementary School was officially introduced following the BNCC (Brasil, 2017), which was promulgated in 2017, given that doctoral theses are developed and published over a period of four years. This is why there was a considerable increase in 2023.

Of these research projects, eight come from education programs and the others from programs involving mathematics and/or science education/teaching. We therefore chose to look at the states. The table below shows the number by state.

Table 7: Distribution of works by state

| State | Quantitative |
|---|---|
| São Paulo | 14 |
| Rio Grande do Sul | 5 |
| Santa Catarina | 3 |
| Pernambuco | 3 |
| Rio de Janeiro | 1 |
| Mato Grosso do Sul | 1 |
| Minas Gerais | 1 |
| Pará | 1 |
| Paraíba | 1 |
| Rio Grande do Norte | 1 |
| Paraná | 2 |

Source: Research data

Table 7 shows that most of the research is concentrated in the state of São Paulo (14), followed by the state of Rio Grande do Sul (5). With this data, we decided to check the university and the supervisor. In the state of São Paulo, most of the research was carried out at the Federal University of São Paulo (3), two of which were supervised by Profa. Dra. Vanessa Dias Moretti, who has carried out research into the development of algebraic thinking with both teachers and students in Elementary School, based on research projects funded by funding agencies such as the São Paulo State Research Foundation (Fapesp), for example.

We also found that two studies were carried out at the University of ABC, under the guidance of Prof. Dr. Alessandro Jacques Ribeiro, who has also produced and disseminated academic research on Algebraic Thinking in Elementary School.

We also found that two more studies were produced at the University of São Francisco, but with different supervisors, one of them by Profa. Dra. Adair Nacarato and the other by Profa. Dra. Daniela Dias dos Anjos.

Against this backdrop, we moved on to our discussions, without losing sight of the objective we had set ourselves.

## 4 Discussions

When we delimited the research corpus, we observed a variety of themes and in order to better organize ourselves, we tried to stick to five aspects, which will be our focus of analysis,

that these studies fit into. To make it easier to see, Table 8 shows the number of studies that focus on these themes.

We then proceeded to the categorization process, thus delimiting these focuses of analysis so as not to lose what each piece of research contributes, thus taking a broader look at each reality represented there and being able to ascertain each specificity of the research sphere.

Table 8: Focus of Analysis

| Focus of Analysis | Researches |
|---|---|
| Initial and continuing training and/or working with teachers<br>Classroom practice<br>Documentary analysis (textbooks and official documents)<br>Technology and algebraic thinking | Initial: Goma (2019); Nascimento (2020) |
| | Initial and ongoing: Santana (2019) |
| | Ongoing: Ferreira (2017); Barboza (2019); Santos (2020); Jungbluth (2020); Silva (2020); Malska (2022); Danielle Silva (2022); Muller (2022); Oliveira (2022); Vanessa Oliveira (2022); Silva (2022); Silva (2023); Romeiro (2023); Quadros (2023) |
| | Other training contexts: Moreira (2020); Conceição (2021) |
| Research mapping | Santos (2017); Jéssica Silva (2019); Mescouto (2019); Marques (2022) |
| Initial and continuing training and/or working with teachers | Teodoro (2020); Gomes (2020); Frasnelli (2021); Medeiros (2021); Souza (2022); Silva (2022) |
| Classroom practice | Souza (2019); Campeão (2020) |
| Documentary analysis (textbooks and official documents) | Oliveira (2022) |

Source: Own elaboration

In view of the table presented, we will begin by explaining the realities dealt with in the research. We focused our efforts on reading each work, emphasizing elements that, in our view, were central so that we could understand how Algebraic Thinking in Elementary School is being approached in the research. By turning to this, we were able to get an overview of what we can call the central elements of our analysis, which are aligned with the general objective of this study.

As the first aspect *Initial and continuing training and/or working with teachers*, we realized from our analysis that there is a growing concern among researchers about teachers' knowledge and the continuing training needed for a better understanding and apprehension of algebraic concepts, given that in their research involving Algebraic Thinking in Elementary School, future teachers and current teachers expressed their difficulties in conceptual understanding of algebra and that this could hinder and influence students' mistakes and understanding of the proposed activities. On the other hand, among the results of the research, we noticed a movement of ascension in thinking, from the abstract to the concrete and it was possible to appropriate theoretical knowledge with the organization of teaching focused on the development of Algebraic Thinking in Elementary School. (Ferreira, 2017; Santos, 2019; Goma, 2019; Barboza, 2019; Nascimento, 2020; Jungbluth, 2020); Oliveira, 2022; Quadros, 2023; Romeiro, 2023).

This concern leads us to wonder whether the initial training of these teachers might not, in a way, be the breeding ground for the development of essential skills for working with these concepts?! It is important to consider that theoretical and practical discussions during initial

training could already address and resolve many of the difficulties that teachers later face in the classroom, promoting more solid and comprehensive preparation. However, we wonder: what are the knowledge and skills of the teacher who will teach mathematics in Elementary School, ensuring that they are well prepared from the start of their career?!

Furthermore, by investigating initial training, we could identify existing gaps and propose improvements that could be implemented in teacher training programs. This would not only benefit future teachers, but would also have a direct positive impact on students, who would receive a more robust and effective Mathematics Education. Thus, it is crucial that our research considers initial training as a vital component for developing teaching skills capable of meeting the challenges of teaching algebraic concepts in Elementary School.

Among the works that focus on continuing teacher training and initial training, we notice that some discuss a particular reality of a group of teachers in a regionalized way and others already point to a possible causality of the facts, looking at the whole and recognizing the teachers' knowledge as being incipient to achieve the goal of developing algebraic thinking in students.

If we look at continuing education, we can only see the end of the process. process, because they are already practicing teachers, in other words, matured by a journey, be it long or short, where it is necessary to make a more practical intervention that can be effective in the reality in which that group finds itself.

The second aspect we observed would be *classroom practices*, which look at pedagogical processes that often aim to ascertain how Algebraic Thinking is being developed in the most diverse educational realities, also permeating the knowledge of some teachers who participate in the research. The key to understanding why there is still little research in this area is the difficulty of correlating exploratory research with practice, but we can see in the works that discuss *classroom practices* a relationship with innovation for the development of algebraic thinking.

It is worth highlighting among these studies (Santos, 2017; Ferreira, 2017; Jessica Silva, 2019; Mescouto, 2019; Marques, 2022), after an analysis, the proposal of innovation and the attempt to bring to the school reality a look at the development of Algebraic Thinking in the light of a perhaps unconventional meaning for teachers, using various pedagogical devices for a convergence between the macro objective and the dimension of day-to-day professional work to move towards these new possibilities. One characteristic that we can see in these studies is the proposal to look at them from the perspective of teaching-learning-evaluation, even though most of them don't make this methodology explicit, we can see a certain regularity in their method of collecting primary data, processing and analyzing this data in their conclusions.

The third aspect, which deals with the *documentary analysis* of Algebraic Thinking from a curricular point of view, shows that there is a concern about how curricula are working with Algebraic Thinking, for example, the correlation between curricular parameters and teachers' knowledge is very relevant, because the analysis of this curriculum or document can interfere with teaching practice itself, revealing the characteristic identity of that teacher.

When we analyze this, in line with this purpose, Elementary School teachers should be those who, instead of focusing their efforts on practicing mechanical exercises with only the reality of the result, should direct their efforts towards developing specific thinking in their students, which most of the time does not occur, thus bringing that pragmatic Mathematics and the teaching of Algebra to the calculation.

The fourth aspect deals with technology aligned with Algebraic Thinking, which we can correlate with the various methodologies used in the field of Education that support the process of developing or maturing Algebraic Thinking in students. Just as we have various ways of using technology to our advantage in everyday school life, we run the risk of losing the real focus of our practice, so, with regard to the use of technology, there is a methodological proposal in the two studies analyzed, one of them using a cultural-historical approach in an attempt to understand the constitution of the subject, based on Vygotsky, in its social relationship, which also touches on the approach we chose to analyze these works, knowing that there is an individual who is located in history at a specific point, with a subjective look at certain realities, making us question methods, certain pre-established ways and our own practice (Souza, 2019; Campeão, 2020).

All of this, together with technological resources, can enable the development of Algebraic Thinking in students in Elementary School, since, at first, they will be instructed in the most diverse social realities, thus confronting their epistemological limits from a socio-historical analysis and within an environment that enhances learning. These new perspectives can provide teachers with a better parameter for developing their lessons in the most diverse realities, so that they can be aware of what is needed for that year of schooling.

The fifth aspect deals with the mapping of research, and as we are analyzing using the same methodology, it is clear to understand the importance of this work, because by delimiting what the research is saying about Algebraic Thinking we will be able to contemplate a good part of what the researchers understand by the theme sought and its nuances.

Mapping allows us to have a comprehensive view, providing us with subsidies so that the appropriation of this theme can contemplate a range of methodological possibilities but also gaps to be studied in future research, as is the case with our research. When we looked at these studies in our corpus, we defined what our objective was, in order to solve our research question, which at the same time gave us a good number of proposals for further study that we couldn't even imagine when we limited our efforts to only research at master's and doctoral level (Oliveira, 2022).

Finally, one aspect that we would like to highlight, but which does not fit into a single indicator, is the production of support material, which leads us to think that if there is work, there is a need for materials that can complement classroom practice. Our research shows that initial and continuing education are the main focus of researchers when it comes to Algebraic Thinking, but this ends up slipping into the other aspects mentioned above.

These discussions led us to some reflections. Will a teacher who leaves his or her first degree to work in a school have the necessary background to tackle the different themes where mathematical thinking will still need fertile soil to be planted? We understand that the answer is negative, or at best, if aligned with support material that has the proposal of truly leading these classes to the process of this development, they will still have mishaps and didactic-methodological difficulties to overcome, which still tells us a lot about initial and continuing education, where we can see a range of possibilities so that the teacher's performance is not trapped or dependent on support material that often hinders the development of thinking and makes thinking algebraically revert to simply operating algebraically, which would be a step backwards.

## 5 Considerations

In face of what has been said in the course of this study, we have observed a growing

number of issues involving Algebraic Thinking in Elementary School in the context of the initial and continuing training of teachers who will teach and who teach Mathematics in Elementary School. New analyses are needed in future research to try to identify these realities, perhaps by looking at the curricular components studied in the initial training courses and the profile of this teacher who will arrive at the school and also at the courses offered to teachers who have already joined a school unit. of this teacher arriving at the school and also the courses offered to teachers who have already joined a school unit.

Considering all these aspects of analysis listed in the course of the research — initial and continuing training and/or work with teachers, classroom practices, document analysis on algebraic thinking, technology aligned with algebraic thinking and research mapping — which is the core of the chosen research, it is worth highlighting the importance of Algebraic Thinking in the context of the mathematical processes that students will carry out during their school years. Based on a process for working on this type of thinking, they will be able to play an autonomous role in interpreting and solving problems in the school sphere, but also in their particular lives and trajectories.

We can see that, yes, there is a great effort on the part of these researchers to make Algebraic Thinking a subject that enters educational spaces as methodological and pedagogical possibilities for these teachers. We know that, from a broad perspective, Algebraic Thinking is discussed and explored in the initial training of these teachers, but when they go out and are confronted with the underlying reality and each particularity, the gap is large in terms of what they know and what they need to develop.

Continuing education, exploratory research, discussions in research groups, seminars, and the like, directly contribute to bringing the fundamental concepts of this thinking into teaching practice, and it is up to us, with a mature look at the issues that guide our training, to be able to lay a concrete foundation and echo the results of research within not only academic circles, which will help so much in the development of our students' mathematical thinking.

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